Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 5: Crossing the Equator

You investigated the relationship between a city's latitude and the length of daylight it experiences throughout the year. You did so by making scatterplots and finding regression models for the functional relationship between the day of the year and the length of daylight for three different cities at three different latitudes in the Northern Hemisphere:

- Houston, Texas—30°N latitude
- Philadelphia, Pennsylvania—40°N latitude
- Winnipeg, Manitoba, Canada—50°N latitude

In this activity, you will investigate the relationship between two cities that are the same distance from the equator, but on opposite sides of it: Houston, Texas, and Porto Alegre, Brazil.

Remember that the data in the tables for this activity describe the length of daylight for the year 2009 for each day. The data table is based on two assumptions:

- The length of daylight is defined as the amount of elapsed time between sunrise and sunset.
- Because 2009 is not a leap year, there are 365 days in the year.

You will need your Summary Table and scatterplots from Student Activity Sheet 4.

1. Porto Alegre, Brazil, is located in the Southern Hemisphere at 30°S latitude. Houston, Texas, is located in the Northern Hemisphere at 30°N latitude. How do you think the graphs of the length of daylight by day would compare for the two cities? Sketch your prediction, if needed, and explain why it might be true.

2. Make a scatterplot of the length of daylight by day in Porto Alegre, Brazil. Plot the points on the same grid that you used for the scatterplots from the previous activity.

Date	Day Number	Houston		Porto Alegre	
		HH:MM	Min.	HH:MM	Min.
Jan. 1	1	10:17	617	14:03	843
Feb. 1	32	10:48	648	13:29	809
March 1	60	11:34	694	12:42	762
Apr. 1	91	12:29	749	11:45	705
May 1	121	13:20	800	10:55	655
June 1	152	13:57	837	10:19	619
July 1	182	14:01	841	10:15	615
Aug. 1	213	13:33	813	10:42	642
Sept. 1	244	12:45	765	11:30	690
Oct. 1	274	11:52	712	12:23	743
Nov. 1	305	11:00	660	13:17	797
Dec. 1	335	10:23	623	13:56	836

Source: U.S. Naval Observatory, www.usno.navy.mil

3. How does the scatterplot for Porto Alegre compare to the scatterplot for Houston? Does this match your prediction? Why do you think this is so?

4. Use your calculator to generate a scatterplot of length of daylight by day for Houston. You may need to re-enter the data into your data lists. In addition, graph the regression equation that you found for Houston.

5. Enter the data for Porto Alegre into a third list and graph both scatterplots on the same screen. Sketch your graph and describe the axes and scaling.

6. Use your calculator to generate a sinusoidal regression model for the Porto Alegre data. Record the equation (round values to the nearest hundredth) in the Summary Table. Factor the value of ***b*** from the quantity (***bx*** – ***c***) and include that form of the equation as well.

7. Graph your model over your scatterplot. How well does the model fit your data?

8. Connect the points on your paper scatterplot with a smooth curve to represent the regression model.

9. How do the regression models for Houston and Porto Alegre compare?

Similarities:

Differences:

10. Use your calculator to determine the maximum and minimum values for the length of daylight in Porto Alegre. Record these ordered pairs in the Summary Table and label them on your scatterplot. To which dates do these values correspond?

11. How does the maximum length of daylight for Porto Alegre compare to the maximum length of daylight for Houston?

12. How does the minimum length of daylight for Porto Alegre compare to the minimum length of daylight for Houston?

13. **REFLECTION:** Based on your observations of Porto Alegre and Houston, what would you conclude about the longest and shortest days for two cities on opposite sides of the equator?

14. Determine the intersection points of the regression models for Houston and Porto Alegre. Mark these points on your scatterplot and record them in your Summary Table.

15. What do the intersection points mean in the context of this situation? ***Hint:*** Recall that your scatterplot shows the ordered pairs (Day Number, Length of Daylight) for Houston and Porto Alegre.

16. How do the intersection points for the graphs of Houston, Philadelphia, Winnipeg, and Porto Alegre compare? What do these points mean in terms of the context of this situation?

17. Suppose you made a scatterplot of the length of daylight by day for Philadelphia (40°N latitude) and San Carlos de Bariloche, Argentina (40°S latitude). Based on what you noticed about the graphs for Houston and Porto Alegre, what would you expect the two scatterplots to look like?

18. **REFLECTION:** What generalization could you make about the relationship between the length of daylight over time for two cities that are the same distance from the equator but on opposite sides of it (like Houston and Porto Alegre)?

19. **EXTENSION:** What would you expect a scatterplot of the length of daylight by day to look like for a city like Quito, Ecuador, which lies on the equator? Why do you think this is so? Use the Internet to find data for Quito and test your conjecture.